Improved Hausdorff dimension estimate for vertical projections in the Heisenberg group

KATRIN FÄSSLER AND RISTO HOVILA

Abstract. We explore the effect of vertical projections on the Hausdorff dimension of sets in the Heisenberg group endowed with the Korányi distance. It is known that the dimension of an at most one-dimensional set generically does not decrease under such mappings. The proof uses a potential-theoretic approach which, for higher dimensional sets, only yields a trivial lower bound. In the present note, we provide an improved estimate for the dimension and thus prove that the previous trivial bound is not sharp. Moreover, for the larger family of projections onto cosets of vertical subgroups, we show that the potential-theoretic approach can be applied to establish almost sure dimension conservation for sets of dimension up to two.

Mathematics Subject Classification (2010): 28A78 (primary); 28A75, 53C17 (secondary).